Lesson Plan 0

Each presenter will have up to 5 minutes for her presentation. The goal is to present the material from each prompt clearly, concisely and completely. Some choices might have to be made in order for a given presentation to fit within the allotted 5 minute window, so each presenter should think carefully about what issues are essential to discuss, and which issues are nice but could be excised if necessary.

A refresher on fields

(1) Present the content of §1 from the text.

(2) Present §1 Exercise 1(b), (e), and (f).

(3) Present §1 Exercise 1(d) and (g).

(4) Prove that if $\mathbb{F}$ is a field and $1 \in \mathbb{F}$ is the element satisfying axiom B3, then either the additive order of 1 is infinite or a prime number. Using your 305 knowledge, use this to define "characteristic" of a field.

(5) Present §1 Exercise 2.

(6) Present §1 Exercise 3.

(7) Present §1 Exercise 5.

A primer on vector spaces

(8) Present the content of §2 from the text.

(9) Present the content of §3, parts (1)–(3) from the text.

(10) Present the content of §3, parts (4)–(6) from the text. [Note that the content of (6) is the claim that examples (1)–(5) can be made more general, and it substantiates this claim by giving a generalization of (5) that works for a general field. Your presentation should provide the generalizations of examples (2) and (4) to an arbitrary field $\mathbb{F}$.]

(11) Let X be a set, V be a vector space, and $\mathbb{F}$ be a field; for concreteness, we'll use $+$ and $\cdot$ to indicate the addition and scaling operations of V. Let $\text{Func}(X, V)$ be the set of functions with domain X and codomain $\mathbb{F}$. Provide natural addition and scaling operations $\oplus$ and $\odot$ so that $\text{Func}(X, V)$ becomes a vector space over $\mathbb{F}$; though you likely won't be able to verify all axioms, be sure to give arguments that verify at least a few axioms.

(12) Suppose that we replace axiom (A3) for a vector space with

(A3') There exists $0 \in V$ so that for all $x \in V$ we have $x + 0 = x$.

while keeping all the other axioms the same. Prove that we could recover axiom A(3) from this (seemingly weaker) system.

(13) Suppose that V is a vector space over a field $\mathbb{F}$. In order to be as precise in our statements as possible, we'll write 0_V and $0_{\mathbb{F}}$ to denote the two relevant zero elements . Prove
   (a) if $x, y, z \in V$ are given with $x + y = x + z$, then $y = z$
   (b) if $x \in V$ is given, then $-1 \cdot x = -x$
   (c) if $\alpha \in \mathbb{F}$ is given, then $\alpha \cdot 0_V = 0_V$

(14) Suppose that V is a vector space over a field $\mathbb{F}$. In order to be as precise in our statements as possible, we'll write 0_V and $0_{\mathbb{F}}$ to denote the two relevant zero elements . Prove
   (a) if $x \in V$ is given, then $0_{\mathbb{F}} \cdot x = 0_V$
   (b) if $\alpha \cdot x = 0_V$, then either $\alpha = 0_{\mathbb{F}}$ or $x = 0_V$