

Chapter 68 Parentheses versus brackets

Old news	New news
<p>Ch15.1 (Amy)</p> <p>If <math>V</math> is an <math>n</math>-dimensional vector space with basis <math>\{x_i\}_{i=1}^n</math> and if <math>\{\alpha_i\}_{i=1}^n</math> are any set of scalars, then there is a unique linear functional <math>y</math> on <math>V</math> such that <math>[x_i, y] = \alpha_i</math> for <math>i=1, \dots, n</math>.</p>	<p>If <math>V</math> is an <math>n</math>-dimensional vector space with basis <math>\{x_i\}_{i=1}^n</math> and if <math>\{\alpha_i\}_{i=1}^n</math> are any set of scalars, then there is a unique <math>y \in V</math> such that <math>(x_i, y) = \alpha_i</math> for <math>i=1, \dots, n</math>.</p>
<p>Ch15.2 (Amy)</p> <p>Let <math>V</math> be an <math>n</math>-dimensional vector space with basis <math>X = \{x_i\}_{i=1}^n</math>. Then there is a uniquely determined basis <math>X' = \{y_i\}_{i=1}^n</math> for <math>V'</math> where <math>[x_i, y_j] = \delta_{ij}</math>.</p>	<p>Let <math>V</math> be an <math>n</math>-dimensional vector space with basis <math>X = \{x_i\}_{i=1}^n</math>. Then there is a uniquely determined basis <math>Y = \{y_i\}_{i=1}^n</math> of <math>V</math> where <math>(x_i, y_j) = \delta_{ij}</math>.</p>

Ch15.3 (Amy)

Let  $V$  be an  $n$ -dimensional vector space. Let  $x \in V$  be non-zero. Then there exists a  $y \in V$  such that  $[x, y] \neq 0$ .

Let  $V$  be an  $n$ -dimensional vector space. Let  $x \in V$  be non-zero. Then there exists a  $y \in V$  such that  $(x, y) = 0$ .

Ch17 (Olivia)  
**Annihilator**

$m^0$

$m^\perp$

Ch44 (Sappa)  
**Adjoints**

$$[Ax, y] = [x, A'y]$$

$$(A+B)' = A'+B'$$

$$(\alpha A)' = \alpha A'$$

$$(AB)' = (B'A')$$

$$(A^{-1})' = (A')^{-1}$$

$$A'' = A$$

\* note:  $A^*$  is a linear transformation on  $V$ .

$$(Ax, y) = (x, A^*y)$$

$$(A+B)^* = A^* + B^*$$

$$(\alpha A)^* = \bar{\alpha} A^*$$

$$(AB)^* = B^* A^*$$

$$(A^{-1})^* = (A^*)^{-1}$$

$$A^{**} = A.$$

Ch45 (Olivia)

**Matrix of Adjoint**

If  $X = \{x_1, \dots, x_n\}$  is any basis in the  $n$ -dimensional vector space  $V$ , if  $X'$  is the dual basis in  $V'$ :

If  $[A; X] = (\alpha_{ij})$ , then

$$[A'; X'] = (\alpha_{ji}).$$

If  $X = \{x_1, \dots, x_n\}$  is any basis in the  $n$ -dimensional vector space  $V$ , if  $Y$  is another basis in  $V$ :

If  $[A; X] = (\alpha_{ij})$ , then

$$[A^*; Y] = (\overline{\alpha_{ji}}).$$

Ch53 (Laura)

**Determinants**

$$\det A' = \det A$$

$$\det A^* = \overline{\det A}$$

Ch54 (Carla)

**Proper values**

Proper values of  $A'$  are the same as those of  $A$ .

Proper values of  $A^*$  are the conjugates of proper values of  $A$ .