Substitution. To make a good substitution, you're looking for integrals that take the form

$$\int f(g(x))g'(x)\ dx.$$

Notice that this means you'll often be looking for a function "trapped" inside another function multiplied by the derivative of that trapped function. You'll often have to introduce constants to make the substitution work out nicely, so don't be afraid to multiply by 1 creatively.

(1) $\displaystyle\int \frac{1}{2+4x}\ dx$

(2) $\displaystyle\int_0^{\pi/5} \sin(5\theta)\ d\theta$

(3) $\displaystyle\int_1^3 \frac{u}{1+u^2}\ du$

(4) $\displaystyle\int \frac{\sin(t)}{\cos^2(t)}\ dt$

(5) $\displaystyle\int x^2 e^{-x^3}\ dx$

(6) $\displaystyle\int_0^1 (u+1)(u^2+2u)^{10}\ du$

(7) $\displaystyle\int \sec^2(x)\ \sqrt[3]{\tan(x)}\ dx$

(8) $\displaystyle\int \frac{1}{x^2}\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)\ dx$

(9) $\displaystyle\int \frac{x\sqrt{\arctan(x^2)}}{1+x^4}\ dx$

Integration by parts. To use integration by parts, you're looking for integrands that look like one function being multiplied by the derivative of another function; i.e., integrals that look like

$$\int f(x)g'(x)\ dx.$$

When the integrand is the product of functions, you'll need to decide which function you should label as $f(x)$ and which should be $g'(x)$. To choose which function is $f(x)$, it's helpful to remember LIATE (preference goes to: logarithms, inverse trig functions, algebraic expressions, trig functions and exponentials). Don't forget that sometimes $g'(x)$ will be disguised as 1!

(1) $\displaystyle\int_0^{\pi} x\cos(2x)\ dx$

(2) $\displaystyle\int xe^{3x}\ dx$

(3) $\displaystyle\int_1^e \ln(x^2)\ dx$

(4) $\displaystyle\int \arctan(x)\ dx$

(5) $\displaystyle\int r^2\ln(r)\ dr$

(6) $\displaystyle\int \sin^2(x)\ dx$

(7) $\displaystyle\int e^{\theta}\sin(\theta)\ d\theta$

(8) $\displaystyle\int_1^{\sqrt{3}} \arcsin\left(\frac{1}{x}\right)\ dt$

Mixed bag. Here are some problems which require both integration by parts and u-substitution.

(1) $\displaystyle\int x^5 e^{x^3}\ dx$

(2) $\displaystyle\int \sin(\sqrt{2x})\ dx$

(3) $\displaystyle\int t^3\ln(1+t^2)\ dt$