Lecture 2 Bonus

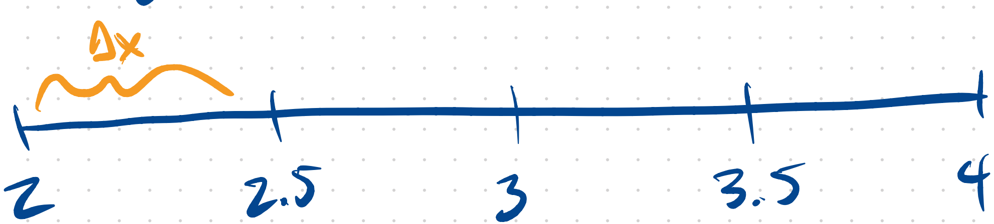
Riemann sum calculations

Ex Approximate $\int_2^4 x^2 - x + 2 \, dx$

using a Riemann sum with $n = 4$ and

using the left-hand rule.

To get started, we'll divide $[2, 4]$ into 4 subint.



$$\Delta x = \frac{4-2}{4} = 1/2$$

Now we need to pick our "sample points" x_i^*

1st interval is $[2, 2.5]$ has $x_1^* = 2$

2nd interval is $[2.5, 3]$ has $x_2^* = 2.5$

3rd interval is $[3, 3.5]$ has $x_3^* = 3$

4th interval is $[3.5, 4]$ has $x_4^* = 3.5$

With this in mind, recall the Riemann sum we want is

$$\underbrace{f(x_1^*) * \Delta x}_{\text{area of first rectangle}} + \underbrace{f(x_2^*) * \Delta x}_{\text{area of second rectangle}} + f(x_3^*) * \Delta x + f(x_4^*) * \Delta x$$

$$= \sum_{i=1}^{4} f(x_i^*) \Delta x$$

Now just plug our f, x_i^*, and Δx in!

From the problem, $f(x) = x^2 - x + 2$. We're already computed $x_1^* = 2$, $x_2^* = 2.5$, $x_3^* = 3$, $x_4^* = 3.5$. We also already knew $\Delta x = 0.5$.

$$f(x_1^*) * \Delta x + f(x_2^*) * \Delta x + f(x_3^*) * \Delta x + f(x_4^*) * \Delta x$$

$$= f(2) * \frac{1}{2} + f(2.5) * \frac{1}{2} + f(3) * \frac{1}{2} + f(3.5) * \frac{1}{2}$$

$$= (2^2 - 2 + 2) * \frac{1}{2} + (2.5^2 - 2.5 + 2) * \frac{1}{2} + (3^2 - 3 + 2) * \frac{1}{2}$$

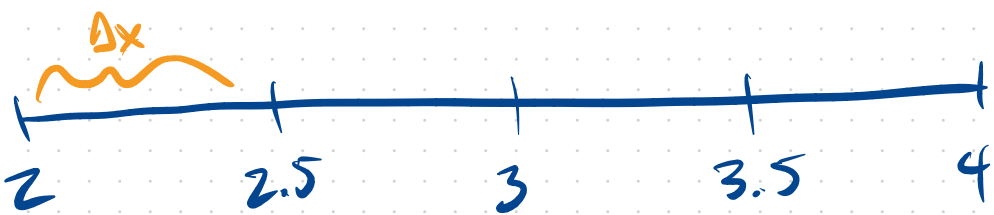
$$+ (3.5^2 - 3.5 + 2) * \frac{1}{2}.$$

Ex Approximate $\int_2^4 x^2 - x + 2 \, dx$

using a Riemann sum with $n = 4$ and

using the midpoint-hand rule.

To get started, we'll divide $[2, 4]$ into 4 subint.



$$\Delta x = \frac{4-2}{4} = 1/2$$

Now we need to pick our "sample points" x_i^*

1st interval is $[2, 2.5]$ has $x_1^* = \frac{2+2.5}{2} = \frac{4.5}{2}$

2nd interval is $[2.5, 3]$ has $x_2^* = \frac{2.5+3}{2} = \frac{5.5}{2}$

3rd interval is $[3, 3.5]$ has $x_3^* = \frac{3+3.5}{2} = \frac{6.5}{2}$

4th interval is $[3.5, 4]$ has $x_4^* = \frac{7.5}{2}$

With this in mind, recall the Riemann sum we want is

$$\underbrace{f(x_1^*) * \Delta x}_{\text{area of first rectangle}} + \underbrace{f(x_2^*) * \Delta x}_{\text{area of second rectangle}} + f(x_3^*) * \Delta x + f(x_4^*) * \Delta x$$

$$= \sum_{i=1}^{4} f(x_i^*) \Delta x$$

Now just plug our f, x_i^*, and Δx in!

From the problem, $f(x) = x^2 - x + 2$. We're already completed $x_1^* = \frac{4.5}{2}$, $x_2^* = \frac{5.5}{2}$, $x_3^* = \frac{6.5}{2}$, $x_4^* = \frac{7.5}{2}$. We also already knew $\Delta x = 0.5$.

$$f(x_1^*) * \Delta x + f(x_2^*) * \Delta x + f(x_3^*) * \Delta x + f(x_4^*) * \Delta x$$

$$= f\left(\frac{4.5}{2}\right) * \frac{1}{2} + f\left(\frac{5.5}{2}\right) * \frac{1}{2} + f\left(\frac{6.5}{2}\right) * \frac{1}{2} + f\left(\frac{7.5}{2}\right) * \frac{1}{2}$$

$$= \left(\left(\frac{4.5}{2}\right)^2 - \frac{4.5}{2} + 2\right) * \frac{1}{2} + \left(\left(\frac{5.5}{2}\right)^2 - \frac{5.5}{2} + 2\right) * \frac{1}{2}$$

$$+ \left(\left(\frac{6.5}{2}\right)^2 - \frac{6.5}{2} + 2\right) * \frac{1}{2} + \left(\left(\frac{7.5}{2}\right)^2 - \frac{7.5}{2} + 2\right) * \frac{1}{2}$$

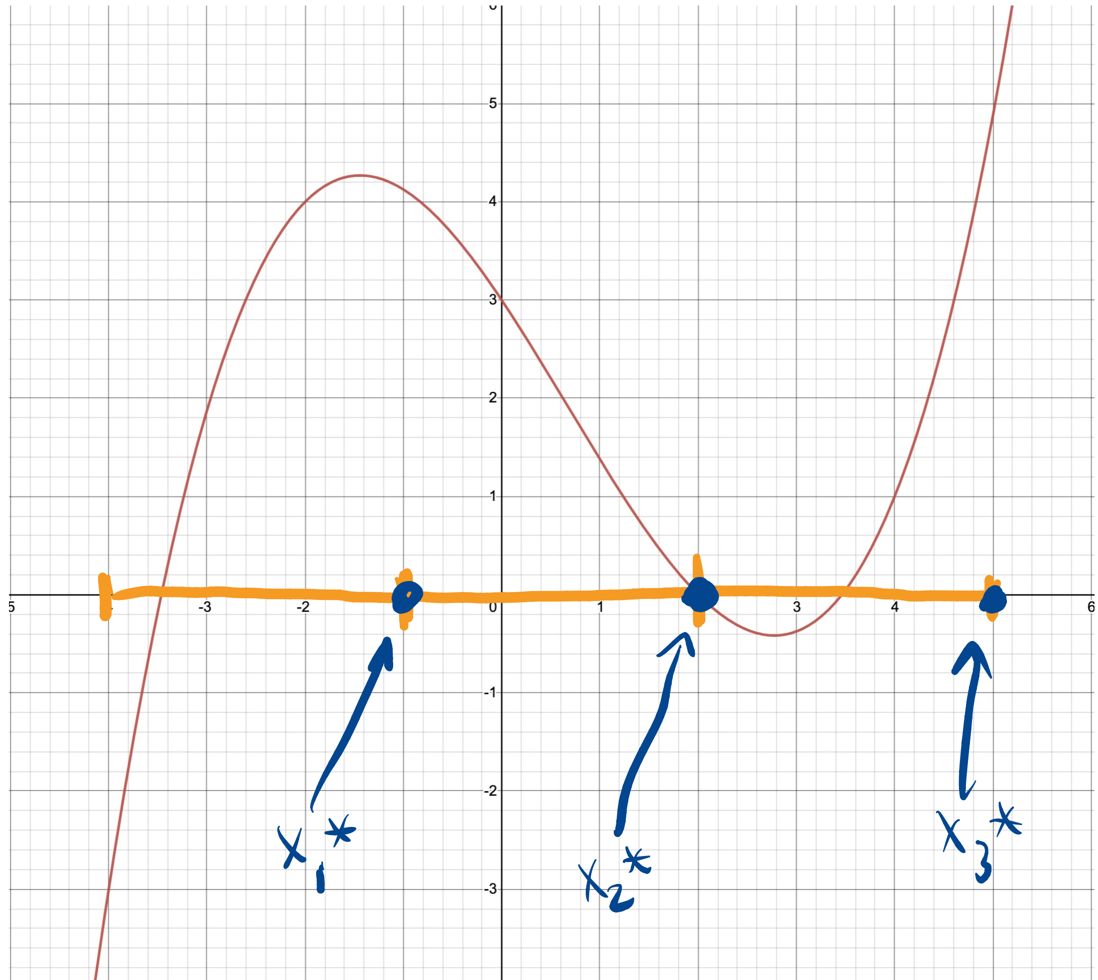
Ex Estimate $\int_{-4}^{5} f(x)\,dx$ for the $f(x)$ depicted $\longrightarrow$ using the right hand Riemann sum with $n=3$

We need to Split $[-4, 5]$ into 3 subintervals of width

$$\Delta x = \frac{5-(-4)}{3} = 3$$

Intervals are

$[-4, -1], [-1, 2], [2, 5]$



The Riemann sum is

$f(x_1^*) * \Delta x + f(x_2^*) * \Delta x$

$+ f(x_3^*) * \Delta x$

$= f(-1) * 3 + f(2) * 3$

$+ f(5) * 3$

$\approx 4.2 * 3 + 0 * 3$

$+ 5 * 3$