Day 9 Bonus

Trig / Inverse Trig
hybrids

In class we computed an integral and our answer involved an expression

$$\tan\left(\operatorname{arcsec}\left(\frac{x}{4}\right)\right)$$

Can we simplify this? Yes!

Key idea: decode what $\operatorname{arcsec}\left(\frac{x}{4}\right)$ means.

To say that $\operatorname{arcsec}\left(\frac{x}{4}\right) = \theta$ means

that $\sec(\theta) = \frac{x}{4}$. We want

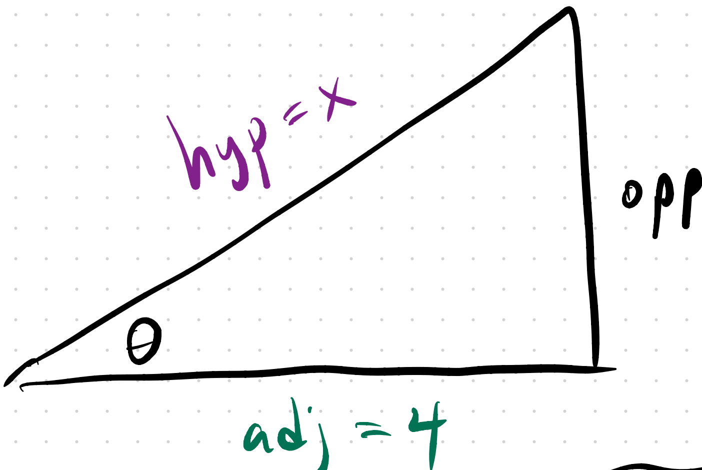
$\tan\left(\operatorname{arcsec}\left(\frac{x}{4}\right)\right) = \tan(\theta)$.

So what do we do:

1. we draw a triangle to "realize" $\sec(\theta) = \frac{x}{4}$
2. Use this drawing to compute $\tan(\theta)$

How to draw a triangle that captures $\sec(\theta) = \frac{x}{4}$?

First, $\frac{x}{4} = \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{adj/hyp} = \frac{hyp}{adj}$



What is $\tan(\theta)$?

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{opp/hyp}{adj/hyp} = \frac{opp}{adj}$$

So: how to compute opp?

Pythag says $opp = \sqrt{x^2 - 4^2}$

$= \sqrt{x^2 - 16}$

Pythagoras says $opp^2 + adj^2 = hyp^2$

$opp^2 + 4^2 = x^2$

Putting all this together, we get

$$\tan\left(\operatorname{arcsec}\left(\frac{x}{4}\right)\right) = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{x^2-16}}{4}$$