June 1

WARM UP PROBLEMS

A) Find $f(x)$ and $g(x)$ so that

$$\cos(e^x) = (f \circ g)(x)$$

B) Suppose a line has slope -2.5 and passes through $(4, 7)$

⇝ What is point-slope form?

⇝ Find A so $(A, 3)$ is on the line.

C) What points are <u>not</u> in the domain of each function?

⇝ $f(x) = \sqrt{x}$

⇝ $g(x) = \dfrac{x^3 - 3x - 2}{x^2 - 4}$

⇝ $h(x) = \sin(x)\cos(x)$

⇝ $k(x) = \ln(x)$

A) Find $f(x)$ and $g(x)$ so that $\cos(e^x) = (f \circ g)(x)$

Reasonable options:

$$\boxed{\begin{aligned} f(x) &= \cos(x) \\ g(x) &= e^x \\ &\Downarrow \\ &\cos(e^x) \end{aligned}}$$

or

$$\begin{aligned} f(x) &= e^x \\ g(x) &= \cos(x) \\ &\Downarrow \\ &e^{\cos(x)} \end{aligned}$$

B) Suppose a line has slope -2.5 and passes through (4,7)

⇝ What is point-slope form?

$$y-7 = -2.5(x-4)$$

$\left(\text{general form:} \quad y-y_0 = m(x-x_0)\right)$

⇝ Find A so (A,3) is on the line

Plug $y=3$ into point slope form and solve for x:

$$3-7 = -2.5(x-4) \rightsquigarrow -4 = -2.5(x-4) \rightsquigarrow \frac{-4}{-2.5} = x-4$$

$$\rightsquigarrow \boxed{x = \frac{-4}{-2.5} + 4}$$

c) what points are <u>not</u> in the domain of each function?

$\rightsquigarrow f(x) = \sqrt{x}$

Domain: all non-negative numbers

interval notation: $[0, \infty)$

$\rightsquigarrow g(x) = \dfrac{x^3 - 3x - 2}{x^2 - 4}$

Domain: everything except ± 2

interval: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

$\rightsquigarrow h(x) = \sin(x)\cos(x)$

Domain: all real numbers

interval: $(-\infty, \infty)$

fancy: $\mathbb{R}$

$\rightsquigarrow k(x) = \ln(x)$

$\ln(x) = y$ means $e^y = x$

Domain: all positive numbers

$(0, \infty)$

New stuff

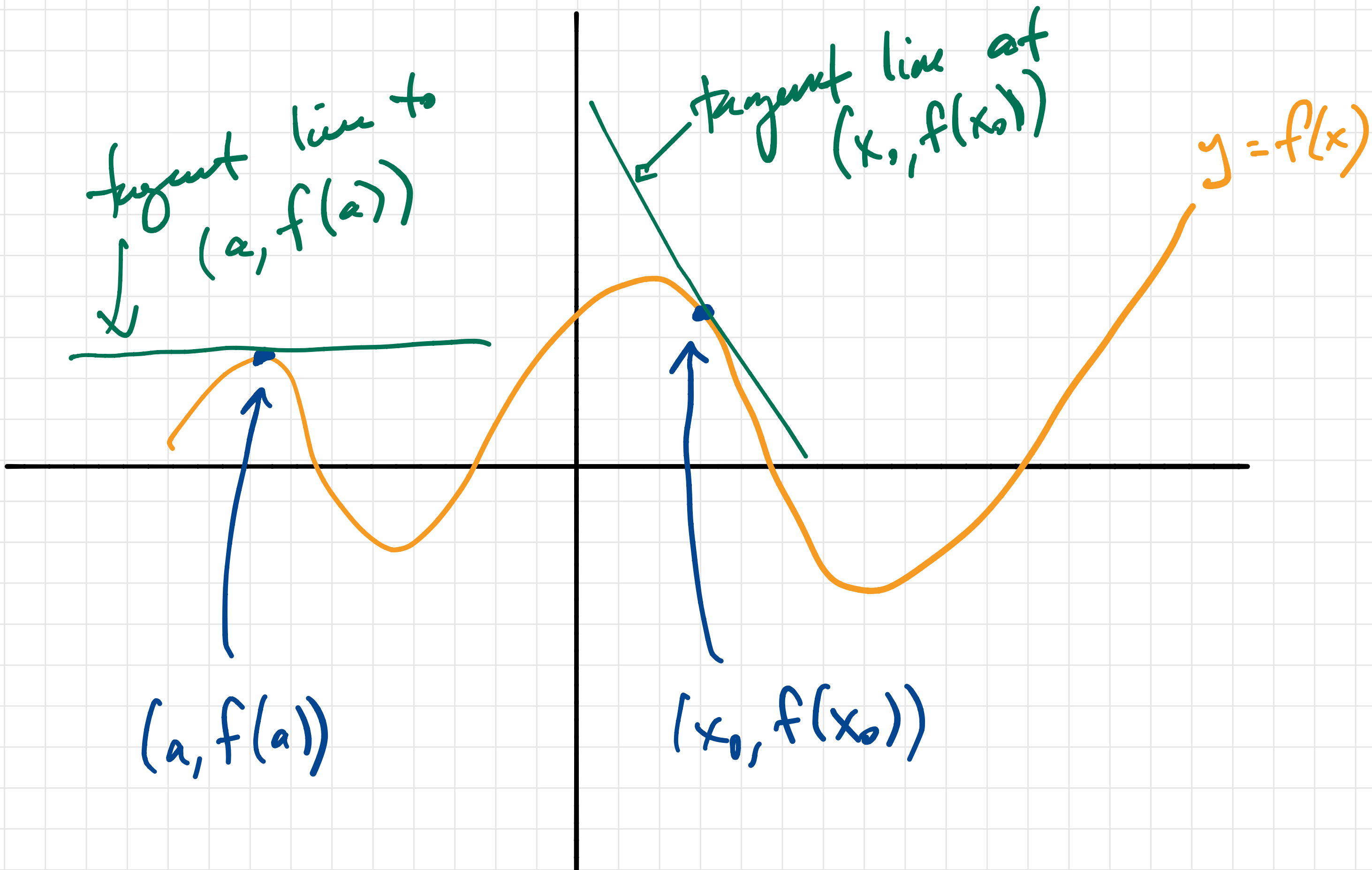
What is calculus?

$\rightsquigarrow$ differential calculus

Big question: for the graph $y = f(x)$ and a point $(x_0, f(x_0))$, what is the "tangent line" to graph?

$\rightsquigarrow$ integral calculus

Ex



To answer the tangent line problem for function $f(x)$ and point $(x_0, f(x_0))$,

we need

① slope of tangent line

② a point on tangent line ✓

The point $(x_0, f(x_0))$ is on the tangent line.

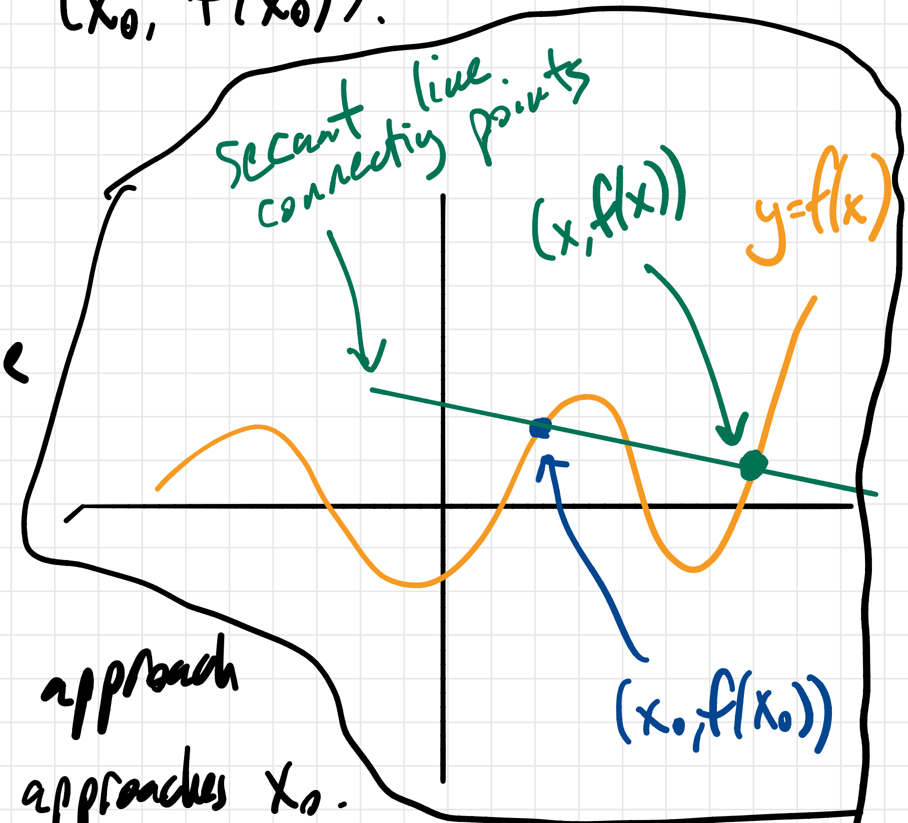
Point slope form : $y - f(x_0) = \boxed{m}(x - x_0)$

???

Our updated goal: find slope of tangent line for $y = f(x)$ at $(x_0, f(x_0))$.

idea to solve this:

if we examine the slope of the secant line connecting $(x_0, f(x_0))$ and $(x, f(x))$, then that should approach slope of the tangent as x approaches x_0.



To summarize, the slope of the tangent line should be the "limit" of the slope of the secant line between $(x_0, f(x_0))$ and $(x, f(x))$ (ie, as x approaches x_0)

$$\text{"} \underbrace{\lim_{x \to x_0}}_{\text{as } x \text{ approaches } x_0} \underbrace{\frac{f(x) - f(x_0)}{x - x_0}}_{\text{slope of secant}} \text{"}$$

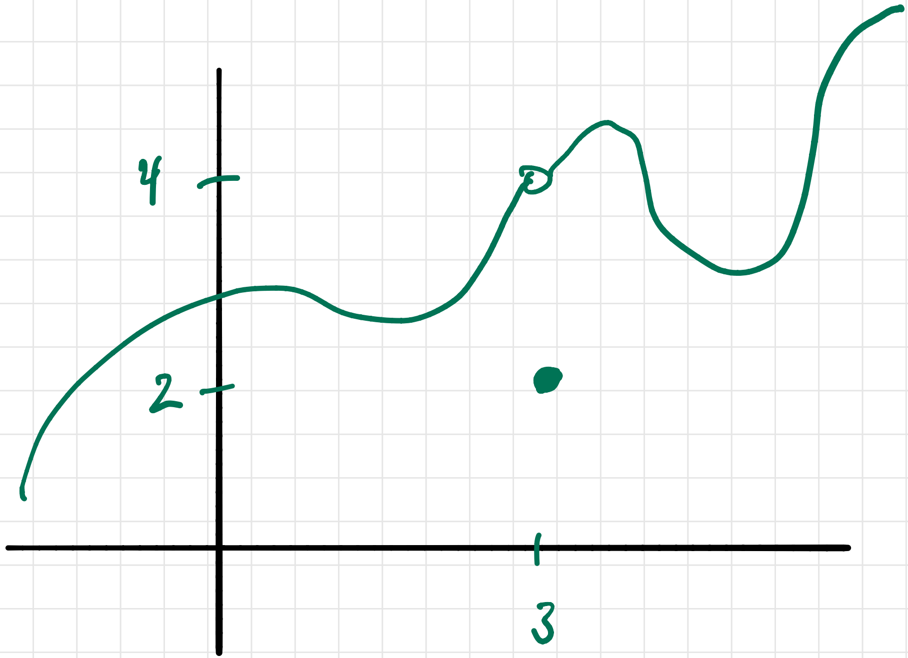
To make sense of this intuition idea, we need to know what limit means.

Intuitive Definition (Limit)

We say $\lim_{x \to a} f(x) = L$ if we can get outputs of $f(x)$ as close to L as we want by making inputs sufficiently close to (but not equal to!) a.

Note: limits don't care about the value of $f(a)$.

Ex Consider $f(x)$ given by



What is

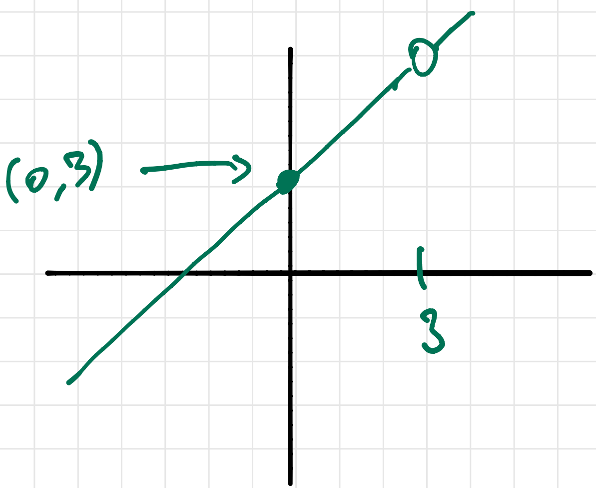
$$\lim_{x \to 3} f(x)?$$

4 since outputs get close to 4 as inputs approach 3.

Ex What is $\displaystyle\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$?

when $x \neq 3$

Note: $\displaystyle\frac{x^2 - 9}{x - 3} = \frac{(x-3)(x+3)}{x-3} = x + 3$



So we get:

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} x + 3$$

$$= 6.$$