

**Substitution.** To make a good substitution, you're looking for integrals that take the form

$$\int f(g(x))g'(x) dx.$$

Notice that this means you'll often be looking for a function "trapped" inside another function, and that you'll also want to ensure that the derivative of that trapped function shows up as a factor in the integrand.

$$(1) \int \frac{1}{2+4x} dx$$

$$(4) \int \frac{\sin(t)}{\cos^2(t)} dt$$

$$(7) \int \sec^2(x) \sqrt[3]{\tan(x)} dx$$

$$(2) \int_0^{\pi/5} \sin(5\theta) d\theta$$

$$(5) \int x^2 e^{-x^3} dx$$

$$(8) \int \frac{1}{x^2} \sec\left(\frac{1}{x}\right) \tan\left(\frac{1}{x}\right) dx$$

$$(3) \int_1^3 \frac{u}{1+u^2} du$$

$$(6) \int_0^1 (u+1)(u^2+2u)^{10} du$$

$$(9) \int \frac{x\sqrt{\arctan(x^2)}}{1+x^4} dx$$

**Integration by parts.** To use integration by parts, you're looking for integrands that look like one function being multiplied by the derivative of another function; i.e., integrals that look like

$$\int f(x)g'(x) dx.$$

When the integrand is the product of functions, you'll need to decide which function you should label as  $f(x)$  and which should be  $g'(x)$ . To choose which function is  $f(x)$ , it's helpful to remember LIATE (preference goes to: logarithms, inverse trig functions, algebraic expressions, trig functions and exponentials). Don't forget that sometimes  $g'(x)$  will be disguised as  $1 dx$ !

$$(1) \int_0^{\pi} x \cos(2x) dx$$

$$(4) \int \arctan(x) dx$$

$$(7) \int e^{\theta} \sin(\theta) d\theta$$

$$(2) \int x e^{3x} dx$$

$$(5) \int r^2 \ln(r) dr$$

$$(8) \int_1^{\sqrt{3}} \arcsin\left(\frac{1}{x}\right) dt$$

$$(3) \int_1^e \ln(x^2) dx$$

$$(6) \int \sin^2(x) dx$$

**Mixed bag.** Here are some problems which require both integration by parts and  $u$ -substitution.

$$(1) \int x^5 e^{x^3} dx$$

$$(2) \int \sin(\sqrt{2x}) dx$$

$$(3) \int t^3 \ln(1+t^2) dt$$