QUIZ 5

Instructions: Complete the following problems. You may use the derivative shortcuts developed in class to answer these problems, though you are encouraged to show as much work as possible so that partial credit may be awarded.

(1) (10 pts) Evaluate $\frac{d}{dx}[\arcsin(x)]$.

In class, we saw that $\frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}$.

(2) (10 pts) Evaluate $\frac{d}{dx}\left[\sqrt[3]{\arctan(x)}\right]$.

Using the chain rule, we have

$$\frac{d}{dx}\left[\sqrt[3]{\arctan(x)}\right] = \frac{1}{3}(\arctan(x))^{-\frac{2}{3}}\frac{1}{1+x^2}.$$

(3) (10 pts) Evaluate $\frac{d}{dx}[x\ln(x) - x]$. Simplify your answer as much as possible.

Using the product rule we have

$$\frac{d}{dx}[x\ln(x) - x] = x\,\frac{1}{x} + \ln(x) - 1 = \ln(x).$$

(4) (10 pts) Evaluate $\frac{d}{dx}\left[\sin(\sqrt{1+x^2})\right]$.

Using the chain rule we have

$$\frac{d}{dx}\left[\sin(\sqrt{1+x^2})\right] = \cos(\sqrt{1+x^2}) \cdot \frac{1}{2\sqrt{1+x^2}} \cdot 2x = \frac{x\cos(\sqrt{1+x^2})}{\sqrt{1+x^2}}.$$

(5) (30 pts) Find $\frac{dy}{dx}$ for the graph $2(x^2+y^2)^2 = 25(x^2-y^2)$.

The chain rule says the derivative of the left-hand side is

$$4(x^2+y^2)(2x+2y\frac{dy}{dx}) = 8x^3 + 8x^2y\frac{dy}{dx} + 8y^2x + 8y^3\frac{dy}{dx},$$

and the derivative of the right-hand side is

$$25(2x-2y\frac{dy}{dx}) = 50x - 50y\frac{dy}{dx}.$$

Combining these equations and solving for $\frac{dy}{dx}$ gives

$$\frac{dy}{dx} = \frac{50x-8x^3-8y^2x}{50y+8x^2y+8y^3}.$$

(6) (30 pts) Believe it or not, cubic polynomials in x and y are often used to encrypt information that travels across the web. This lets you and I order calculus books off of Amazon.com without fear that someone will steal our credit card information. One of the steps in the encryption is to find the equation of the line tangent to a given cubic, something we're quite proficient at doing! Here's a problem that asks you to perform this kind of calculation.

Use your calculus skills to find the equation of the line tangent to the graph $x^2 = y^3 + y^2$ at the point $(\sqrt{2}, 1)$.

To find the tangent line I need to know it's slope, and I know from class that this slope is given by the quantity $\frac{dy}{dx}$ evaluated at $(\sqrt{2}, 1)$. So I start by computing $\frac{dy}{dx}$ using implicit differentiation. The derivative of the left hand side is simply $\frac{d}{dx}\left[x^2\right] = 2x$, and the derivative of the right hand side is

$$\frac{d}{dx}\left[y^3 + y^2\right] = 3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} = (3y^2 + 2y)\frac{dy}{dx}.$$

I can now solve:

$$\frac{dy}{dx} = \frac{2x}{3y^2 + 2y}.$$

Hence the slope of the tangent line at $(\sqrt{2}, 1)$ is

$$\frac{2\sqrt{2}}{3+2} = \frac{2\sqrt{2}}{5}.$$

Hence, using the point slope equation, the tangent to the graph at $(\sqrt{2}, 1)$ is given by

$$y - 1 = \frac{2\sqrt{2}}{5}\left(x - \sqrt{2}\right).$$

If you finish early, use this space to list your favorite TV shows today. Compare and contrast these shows with *Full House*, your favorite television show when you were 8.