MIDTERM 1 (PRACTICE)

- Complete the following problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted.
- In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- Please sign the following:

  "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

  Signature: _____

The following boxes are strictly for grading purposes. Please do not mark.

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(1) Complete each of the following sentences.
   (a) The statement of the intermediate value theorem is

   (b) The derivative of a function $f(x)$ at a point a is defined to be

   (c) For a function $f(x)$, we defined $\lim_{x\to a} f(x)$ to be

(2) Determine whether each statement is true or false for arbitrary functions $f(x)$ and $g(x)$. **If the statement is true, cite your reasoning. If it is false, provide a counterexample.**

(a) If $\lim_{x\to 0} g(x) = 0$, then $\lim_{x\to 0} \dfrac{f(x)}{g(x)}$ does not exist.

(b) If $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ exist, then $\lim_{x\to 0}[f(x)g(x)]$ exists.

(3) Give an example of functions f and g so that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, while

$$\lim_{x\to 0}[f(x)+g(x)]$$

does exist. Explain why your functions satisfy the given conditions.

(4) Let $f(x) = 3^{x^3+3}$.

(a) Find functions $g(x), h(x)$, and $i(x)$ so that $f(x) = (g \circ h \circ i)(x)$.

(b) Compute $f^{-1}(x)$.

(c) Compute $\lim_{x \to 1} f(x)$, justifying your conclusion.

(5) Compute

$$\lim_{u \to -1} \frac{\sqrt{5+u} - 2}{u+1},$$

if it exists. If it does not exist, explain why.

(6) Let $f(x) = x^2 + 1$.

(a) The slope of the tangent line passing through the point $(1, 2)$ is given by a particular limit. Write this limit.

(b) Compute the limit in part (a) above.

(c) Use your result from (b) to write the equation for the tangent line to $f(x)$ at the point $(1, 2)$.

(7) Show that there is a solution to the equation $\log_2(x+1) + \sqrt{x} = 1$ in the interval $(0, 1)$.