SI Session #8 | March 19, 2008
Sections 15.6-15.7 | Math 205-03

15.6: Directional Derivatives and the Gradient Vector

Definition of directional derivative:

$D_{\mathbf{u}}f(x,y) = \nabla f(x,y) \cdot \mathbf{u}$

$D_{\mathbf{u}}f(x,y) = |\nabla f(x,y)| \cos\theta$

Warm-up. Find the directional derivative of $f(x,y) = x^2 + y^3 - xy$:

A. In the direction of $\mathbf{v} = \langle -1,3 \rangle$.

B. In the direction indicated by the angle $\theta = 2\pi/3$ at the point (0,1).

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<u>Example.</u> Suppose you are climbing a hill whose shape is given by the equation

$$z = (-1/5)x^2 - y^2/x^2 + 5x$$

where x, y, z are measured in meters, and you are standing at a point with coordinates (5,10,16). The positive x-axis points east and the positive y-axis points north.

A. If you walk northwest, will you start to ascend or descend, and at what rate?

B. If you walk in the direction toward the point (6,8), will you start to ascend or descend, and at what rate?

C. In which direction is the slope the largest? What is the rate of ascent in this direction? At what angle above the horizontal does the path in that direction begin?

In general:

If you are standing at the top of a mountain at a point (*a*,*b*) on a curve *z* = *f*(*x*,*y*)…	
Question	**How to Solve**
How would you calculate the rate of ascent in a particular compass direction (assuming it lies on a specific coordinate axis)?	
How would you calculate the rate of ascent toward a point (*x*,*y*)?	
In which direction is the slope the largest and what is its rate of ascent?	
What is the direction of steepest descent and its respective rate?	
What direction must you travel in order to continue on a flat plane?	
How do you find the angle you must travel in order to travel a particular direction?	

15.7: Maximum and Minimum Values

Committing the Second Derivative Test to Memory:

1. Calculate $D = f_{xx}f_{yy} - f_{xy}^2$.
2. Evaluate $D(a,b)$ for the point you wish to investigate.

Calc I Second Derivative Test for a critical point *c*		**Calc III Second Derivative Test for a critical point (*a,b*)**		
$f''(c) > 0$	local minimum	$D(a,b) > 0$	$f_{xx}(a,b) > 0$	local minimum
$f''(c) < 0$	local maximum	$D(a,b) > 0$	$f_{xx}(a,b) < 0$	local maximum
---	---	$D(a,b) < 0$		saddle
$f''(c) = 0$	inconclusive	$D(a,b) = 0$		inconclusive

Example 1. Let $f(x,y) = 2 + xy - x^2$.

A. Find and classify the critical points of *f*.

B. Find the maximum and minimum values of *f* within the triangle formed by the coordinates (0,-2), (1,-2), and (1,0).

Example 2. Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point (4,2,0).

Example 3. (This is exactly like #2!) Find three positive numbers whose sum is 12 and the sum of whose squares is as small as possible.