Integration Review

Definition of the integral:

Let x_i, $0 \le i \le n$, be the endpoints of n equal subdivisions of the interval $[a, b]$, with $x_0 = a$ and $x_n = b$. Let Δx be the length of each subdivision, so $\Delta x = (b - a)/n$. For each $1 \le i \le n$, let x_i^* be any point in the subinterval $[x_{i-1}, x_i]$. Also let $f(x)$ be continuous on $[a, b]$. Then *the integral of* $f(x)$ *from* a *to* b is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

Basic indefinite integrals (C *is any constant):*

$$\int x^a\,dx = \frac{1}{a+1}x^{a+1} + C, \quad a \ne -1$$

$$\int a^x = \frac{1}{\ln a}a^x + C, \quad a > 0$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \quad \text{(special case of above)}$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \tan x \sec x\,dx = \sec x + C$$

$$\int \cot x \csc x\,dx = -\csc x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

$$\int \frac{-1}{\sqrt{1-x^2}}\,dx = \arccos x + C$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$

$$\int \frac{-1}{1+x^2}\,dx = \operatorname{arccot} x + C$$

$$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \operatorname{arcsec} x + C$$

$$\int \frac{-1}{x\sqrt{x^2-1}}\,dx = \operatorname{arccsc} x + C$$

$$\int \cosh x\,dx = \sinh x + C$$

$$\int \sinh x\,dx = \cosh x + C$$

Properties of integrals:

Linearity: $\int (f(x) + g(x))\, dx = \int f(x)\, dx + \int g(x)\, dx$

$\int cf(x)\, dx = c \int f(x)\, dx, \quad c$ any constant

Switching limits: $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

Breaking up integrals: If c is any point in $[a, b]$, then $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

Comparison properties: If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x)\, dx \geq 0$

If $f(x) \leq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)\, dx \leq \int_a^b g(x)\, dx$

Fundamental Theorem of Calculus:

Part I: Suppose $f(x)$ is continuous on $[a, b]$. Then

$$\frac{d}{dx} \int_a^x f(t)\, dt = f(x).$$

Part II: Suppose $f(x)$ is continuous on $[a, b]$ and suppose $F(x)$ is any antiderivative of $f(x)$ (i.e. $F'(x) = f(x)$). Then

$$\int_a^b f(x)\, dx = F(b) - F(a).$$