

☛ Section 15.1: Functions of Several Variables**Important concepts:**

1. Finding and sketching the domain and/or range of a function
2. Sketching the graph of simple equations
 - a. Planes
 - b. Hemispheres
 - c. Cylinders
 - d. Not-too-bad wave-things with cosines
3. Drawing & understanding contour maps

☛ Section 15.2: Limits and Continuity**Important concepts:**

1. Showing a limit does not exist
2. Determining the values at which a function is continuous

Example 1. Determine the set of points at which $g(f(x,y))$ is continuous.

$$g(t) = t + \ln t \quad f(x,y) = \frac{1-xy}{1+x^2y^2}$$

Example 2. Determine the limit of the following function as $(x,y) \rightarrow (0,0)$, if it exists, or prove that the limit does not exist:

$$f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$$

Example 3. Determine the limit of the following function as $(x,y) \rightarrow (1,-1)$ if it exists, or prove that it does not exist:

$$f(x,y) = \frac{5 - 2x^2 + 3xy}{3 - \sqrt{2x^2 - 3xy + 4}}$$

📌 Section 15.3: Partial Derivatives

Important concepts:

1. Computing f_x and f_y of a function
2. Computing all second derivatives of a function
3. Verifying that Clairaut's Theorem holds

Example 1. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $z = f(x+y)$.

See additional handout for some examples involving partial derivatives.

📌 Section 15.4: Tangent Planes and Linear Approximations

Important concepts:

1. Finding the equation for a tangent plane to a given point
2. Showing a function is differentiable at a given point

Reminder: When is a function **differentiable** (without using the definition of differentiability)?

Example 1. How would we show that $f(x,y) = \sin(2x + 3y)$ is differentiable at $(-3, 2)$?

Example 2. Find the linear approximation for $z = y \ln x$ at $(1,4,0)$.

🔗 Section 15.5: The Chain Rule

Important concepts:

1. Knowing which chain rule to use
2. Being able to apply each of the chain rules
3. Understanding implicit differentiation

	Chain Rule #1	Chain Rule #2
When do we use it?		
What does it say?		
What does it say for $w = F(x, y, z)$?		
An example in which you would use this chain rule		

Example 1. Find dw/dt when:

$$w = \ln\sqrt{x^2+y^2+z^2} \quad x = \sin t, \quad y = \cos t, \quad z = \tan t$$

Example 2. Find $\frac{\partial z}{\partial t}$ when $z = e^{(x+y)}$, $x = s/t$, and $y = t/s$.

Example 3. Find $\frac{\partial z}{\partial x}$ when $xyz = \cos(x + y + z)$. What's the formula for finding $\frac{\partial z}{\partial y}$?