Name: ___________________________________
Math 205-03: Sections 16.8, 17.1-2
April 28, 2008

✣ Section 16.8: Triple Integrals in Spherical Coordinates

➤ Conversions

Cylindrical ↔ Rectangular

Cylindrical	→	Rectangular
$r\cos\theta$	→	x
$r\sin\theta$	→	y
z	→	z

Rectangular	→	Cylindrical
$\text{sqrt}(x^2+y^2)$	→	r
$\text{Tan}^{-1}(y/x)$	→	θ*
z	→	z

* Pay attention to which quadrant θ is in; the value of θ depends on the values of x and y.

Spherical ↔ Rectangular

Spherical	→	Rectangular
	→	x
	→	y
	→	z

Rectangular	→	Spherical
	→	ρ
	→	θ*
	→	$\phi^{\dagger}$

* Pay attention to which quadrant θ is in; the value of θ depends on the values of x and y.
† Note that $0 \le \phi \le \pi$.

Cylindrical ↔ Spherical

Cylindrical	→	Spherical
$\text{sqrt}(r^2+z^2)$	→	ρ
θ	→	θ
$\text{Tan}^{-1}(r/z)$	→	ϕ*

Spherical	→	Cylindrical
$\rho\sin\phi$	→	r
θ	→	θ
$\rho\cos\phi$	→	z

* Note that $0 \le \phi \le \pi$.

Example 1. (a) Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane, and below the cone $z = \text{sqrt}(x^2 + y^2)$.

(b) Set up an integral to find the z-coordinate of the centroid of this solid.

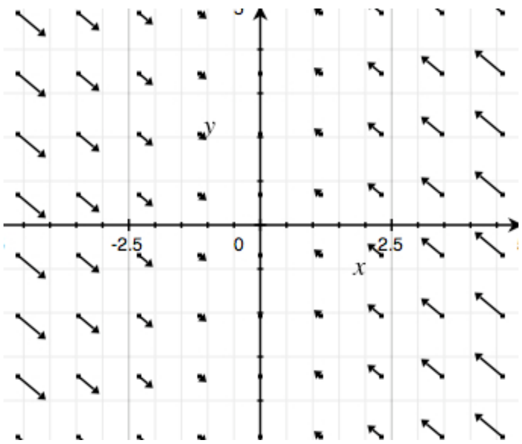
Example 2. Convert the following integral in rectangular coordinates into an integral expressed in spherical coordinates:

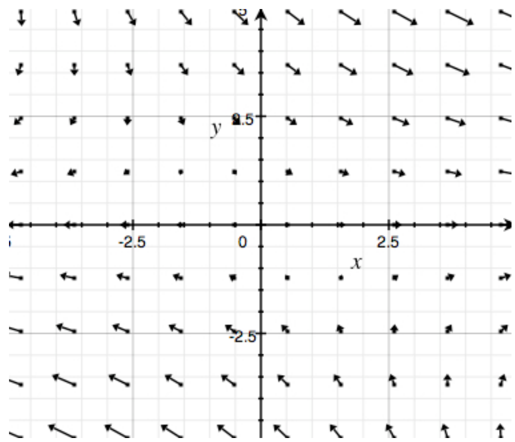
$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} x^2 z \, dz \, dy \, dx$$

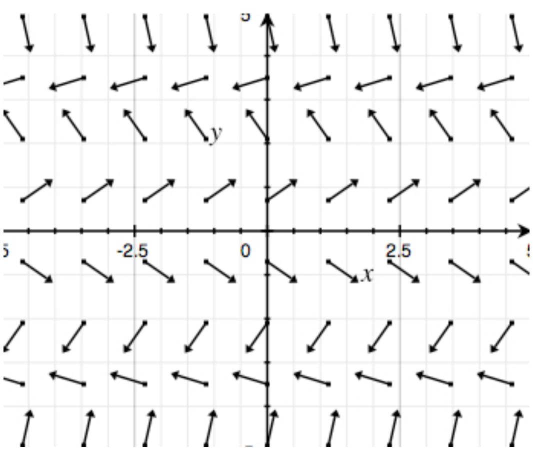
✣ Section 17.1: Vector Fields

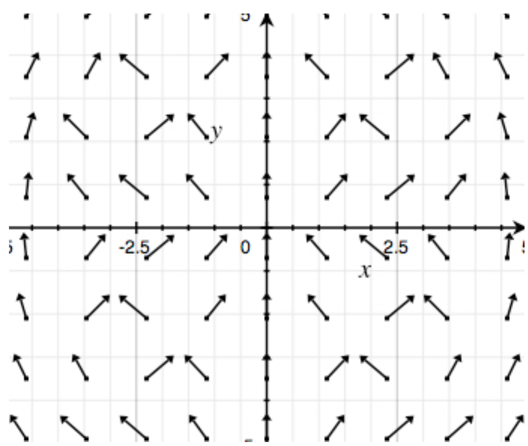
Match the following vector fields with their corresponding graphs:

1. **F**$(x,y) = \langle -x, x \rangle$
2. **F**$(x,y) = \langle \cos y, \sin y \rangle$
3. **F**$(x,y) = \langle \sin(xy), 1 \rangle$
4. **F**$(x,y) = \langle x+y, -y \rangle$









Tricks in Drawing & Identifying Vector Fields:

✣ Section 17.2: Line Integrals

➤ Evaluating Line Integrals

Example 1. Evaluate $\int_C x\, e^y\, dx$, where C is the arc of the curve $x = e^y$ from (1,0) to (e,1).

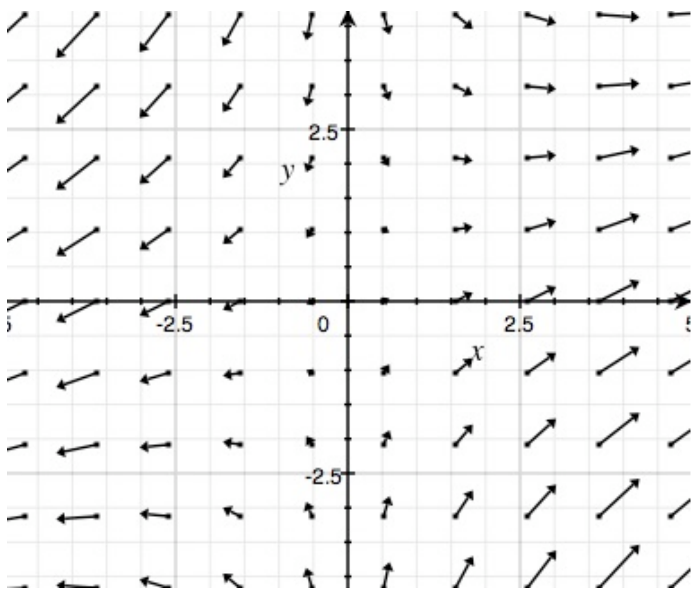
Example 2. Evaluate $\int_C z\, dx + x\, dy + y\, dz$, where C: $x = t^2$, $y = t^3$, $z = t^2$, $0 \le t \le 1$

➤ Interpreting Work & Vector Fields

Definition:

Work = the line integral of **F** along $C = \int_C \mathbf{F} \bullet d\mathbf{r} = \int_C \mathbf{F}(\mathbf{r}(t)) \bullet \mathbf{r}'(t)\, dt$

Example. Let **F** be the the vector field shown below.
(a) Draw and label a curve C_1 in which the line integral of **F** along C_1 is positive.
(b) Draw and label a curve C_2 in which the line integral of **F** along C_2 is negative.



➤ Evaluating Work

Example. Evaluate the line integral $\int_C \mathbf{F} \bullet d\mathbf{r}$, where $\mathbf{F}(x,y,z) = z\,\mathbf{i} + y\,\mathbf{j} - x\,\mathbf{k}$, and $\mathbf{r}(t) = t\,\mathbf{i} + \sin t\,\mathbf{j} + \cos t\,\mathbf{k}$, $0 \leq t \leq \pi$.