

Name: _____

February 13, 2008

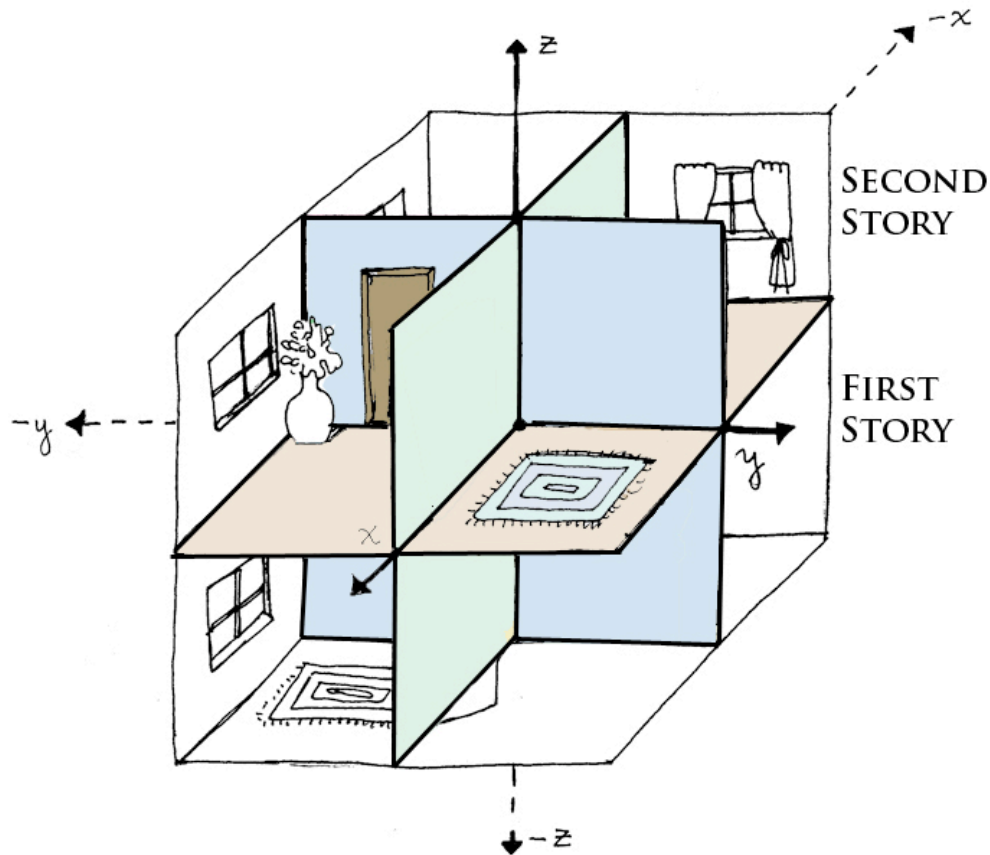
Math 205 - 03 - SI Session #3

Sections 13.4 & 13.5

Visualizing \mathbb{R}^3 (aka 3-Space)

➤ The Coordinate Planes & the Octants

(The animation can be found at: <http://starsabove.org/205/si4.dir>)



In which rooms are values of x positive?

... y positive?

... z positive?

➤ Cross Products

Definition: $\mathbf{a} \times \mathbf{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle$

Geometric interpretation:

Parallelepipeds:

Example 1: Find the area of the parallelepiped given by the vectors $\mathbf{a} = \langle 3, -1, 2 \rangle$, $\mathbf{b} = \langle 1, 3, -3 \rangle$, and $\mathbf{c} = \langle 4, 1, -1 \rangle$.

Example 2: Find the area of the parallelepiped given by the vectors $\mathbf{u} = \langle 1, 1, 1 \rangle$, $\mathbf{v} = \langle 1, 2, 3 \rangle$, and $\mathbf{w} = \langle -1, 0, 1 \rangle$.

Coplanarity: Three vectors lie in the same plane (i.e. are *coplanar*)

⇔ _____

So the vectors from Example # ____ are coplanar.

Torque: a vector (denoted τ) that measures a force's tendency to move an object about an axis

- Examples:

1. Spinning the seconds arrow around a clock involves torque
2. Using a wrench around a bolt involves torque

- Figuring out torque requires:

1. A force vector
2. How big the object you're turning is
3. How much you've turned the object

Recall: $|\mathbf{a} \times \mathbf{b}| =$

Formula: $|\tau| =$

Example 3: A 10 cm wrench is tightening a bolt by applying a 20-N force and rotating it 30 degrees. What's the magnitude of the torque around the bolt?

➤ Lines in Three Dimensions

Two lines are **parallel** \Leftrightarrow _____

Examples:

1. $\mathbf{a}(t) = \langle 1, 3, 4 \rangle + t \langle 2, 3, 6 \rangle$ and $\mathbf{b}(t) = \langle 0, 1, 3 \rangle + t \langle 3, 9/2, 9 \rangle$

2.

Two lines **intersect** \Leftrightarrow (i) _____ and

(ii) _____

Example: Find the point of intersection between these two lines.

Line 1: $\mathbf{a}(t) = \langle 1, 1, 1 \rangle + t \langle 2, 3, 6 \rangle$

Line 2: $\mathbf{b}(t) = \langle 1, 2, 3 \rangle + t \langle 1, 0, -1 \rangle$

Two lines are **skew** \Leftrightarrow (i) _____ and

(ii) _____

Example: Show these two lines are skew.

Line 1: $x = t;$

$$y = 1 + 2t;$$

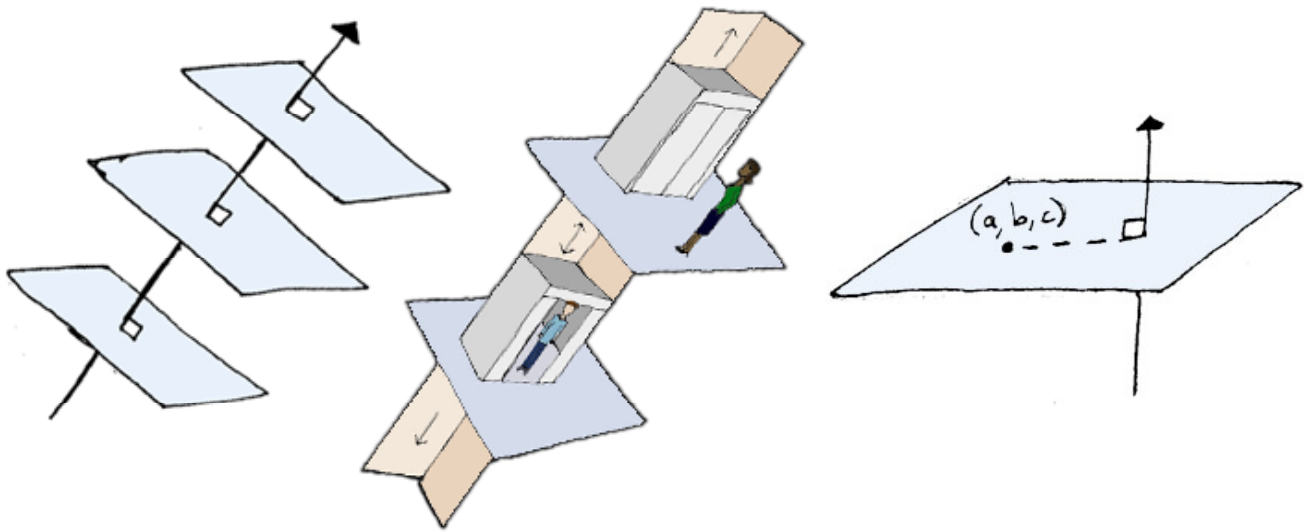
$$z = 2 + 3t$$

Line 2: $x = 3 - 4s;$

$$y = 2 - 3s;$$

$$z = 1 + 3s$$

➤ Understanding Planes



To determine a plane, we need:

(1) _____, analogous to the _____

(2) _____, analogous to the _____

Vector form:

Scalar form:

Parametric form: