Math 223, Spring 2009
Second Midterm Exam, April 23, 2009

Name: ____________________________ Student ID: _________________________

Directions: Check that your test has 10 pages, including this one and the blank one on the bottom (which you can use as scratch paper or to continue writing out a solution if you run out room elsewhere). Please answer all questions and **show all your work**. **Write neatly: solutions deemed illegible will not be graded, so no credit will be given.** This exam is closed book, closed notes, and no calculators are allowed. You have 70 minutes. Good luck!

1. (6 points) ______________
2. (6 points) ______________
3. (5 points) ______________
4. (7 points) ______________
5. (7 points) ______________
6. (8 points) ______________
7. (10 points) ______________
8. (7 points) ______________

Total (out of 56): ______________

Total after the curve (out of 100): ______________

Exam letter grade: ______________

1. (2 pts each) Write precise definitions of the following. Please write in full sentences.

   (a) Mersenne prime

   (b) Perfect number

   (c) Primitive root modulo prime p (either definition)

2. (2 pts each) State the following theorems.

   (a) Dirichlet's Theorem on Primes in Arithmetic Progression

   (b) Euclid's Perfect Number Formula

   (c) Primitive Root Theorem

3. (5 pts) Prove that there are infinitely many primes.

4. (7 pts) Prove Order Divisibility Property.

5. (7 pts) 9 monkeys store their bananas in 7 piles of equal size with the eighth pile of 3 left over. When they divide the bananas into 9 equal groups, 5 remain. What is a possible number of bananas they could have? Answers found by inspection will receive no credit.

6. (4 pts each)

   (a) How many primitive roots modulo 11 are there? Justify your answer. Answers found by inspection will receive no credit.

   (b) Given that 2 is a primitive root modulo 11, find all the other primitive roots modulo 11 (you do not need to reduce your answer mod 11). Answers found by inspection will receive no credit.

7. (10 pts) Let m be a positive integer such that $\phi(m) = 480$. Find a positive integer a such that

$$a \equiv 23^{482} \ (m).$$

8. (7 pts) If p and q are odd primes, prove that pq cannot be a perfect number.